HW 2.

prove: f : $\mathbb{R} - > \mathbb{R}$ defined by $f(x)$ = $\frac{1}{1+x^2}$ is uniformaly continuous.

Recall: $f : A - > \mathbb{R}$ is uniformly conf. if $\forall \epsilon > 0, \exists \delta > 0$ s.t. $\forall x, y \in A$ with $|x - y| < \delta$, we have $|f(x) - f(y)| < \epsilon$

Proof: Let $\epsilon > 0$. Choose $\delta = \frac{\epsilon}{2}$. Then $\forall x, y \in \mathbb{R}$ with $|x - y| < \delta$, $\left| \frac{1}{1+x^2} - \frac{1}{1+y^2} \right| < \delta$

$\left| \frac{1}{1+x^2} - \frac{1}{1+y^2} \right|$ = $\left| \frac{1+y^2-(1+x^2)}{(1+x^2)(1+y^2)} \right|$ = $\frac{|y^2-x^2|}{|1+x^2||1+y^2|}$ = $\frac{|x^2-y^2|}{|1+x^2||1+y^2|}$ = $\frac{|x+y||x-y|}{|1+x^2||1+y^2|}$ = $(\frac{|x+y|}{|1+x^2||1+y^2|}) |x - y| \leq$ $(\frac{|x|}{|1+x^2||1+y^2|} + \frac{|y|}{|1+x^2||1+y^2|}) |x - y| \leq (\frac{|x|}{|1+x^2|} + \frac{|y|}{|1+y^2|}) |x - y|$ < $(1 + 1) |x - y|$ = $2 |x - y|$ < 2δ = $2(\frac{\epsilon}{2})$ = ϵ

Claim $\frac{|a|}{|1+a^2|} \leq 1$,

If $a \leq 1$, $\frac{|a|}{|1+a^2|} \leq \frac{1}{|1+a^2|} \leq \frac{1}{1} = 1$

If $a > 1$, $\frac{|a|}{|1+a^2|} \leq \frac{|a|}{|a^2|} = \frac{1}{|a|} = \frac{1}{a} < 1$

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