HW2.

정보보호공학과 01921061 황송이

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1 Continuity and Uniform Continuity

f is continuous at $\hat{x}$ iff for any convergent sequence $\{x_i\}_{i\in\mathbb{N}}$ to $\hat{x}$, $f(x_i)$ also converges to $f(\hat{x})$.

1.1 Example

$f : \mathbb{X} \to \mathcal{M}$ is continuous, and $\mathbb{X}$ is compact then f is uniform continuous on $\mathbb{X}$

1.2 Proof

Since f is continuous, $\forall x \in \mathbb{X}$, $\forall \delta > 0$, $\exists \epsilon_x > 0$ such that $\| f(y) - f(x) \| < \delta$ implies that $y \in B^0(x, \epsilon_x)$. Since $\mathbb{X}$ is compact, $\{B^0(x, \epsilon_x)\}_{x\in\mathbb{X}}$ covers $\mathbb{X}$ (i.e. $\bigcup_{x\in\mathbb{X}} B^0(x, \epsilon_x) \supset \mathbb{X}$) Moreover, Since $\mathbb{X}$ is compact, there exists a finite number of open covering, say $\{x^i\}_{i=1}^k$ such that $\bigcup_{i=1}^k B^0(x_i, \epsilon_x) \supset \mathbb{X}$. Then, pick ϵ such that $\epsilon = \min_{i=\{1\cdots k\}} \epsilon_{x_i}$. It implies that ϵ satisfies the property.

2 Twice Differentiable

$g''(s) = \left\langle (y-x), \frac{d^2 f(x+s(y-x))}{d x^2}(y-x) \right\rangle$

2.1 Example

Then, Set $(1-s)g''(s) = \frac{d}{d s}((1-s)g'(s) + g(s))$

2.2 Proof

$$
\begin{aligned}
\int_0^1 (1-s)g''(s) d s &= \int_0^1 \frac{d}{d s}((1-s)g'(s) + g(s)) d s \\
&= \int_0^1 d((1-s)g'(s) + g(s)) \\
&= 0 \cdot g'(s) + g(1) - g'(s) - g(0) \\
&= g(1) - g(0) - g'(s) \\
&= f(y) - f(x) - \langle \nabla f(x), y - x \rangle
\end{aligned}
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