

HW4

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1 Exercise 2

Consider the function $f(x) = x^2e^{-x} - x$ With $x \in \mathbb{R}$. Determining the behavior of the Armijo gradient method on this function when $x_0 = 0.5$ and $x_0 = 2$.

1.1 Step 1 - Compute the Descent Direction

$$h_i = h(x_i) = -\nabla f(x_i), \text{ stop if } \nabla f(x_i) = 0$$
$$\nabla f(x) = -x^2e^{-x} + 2e^{-x}x - 1$$

1.2 Step 2 - Compute the Step Size rule

$$\lambda_i = \lambda(x_i) \triangleq \arg \max_{k \in \mathbb{N}} \{ \beta^k | f(x_i + \beta^k h_i) - f(x_i) \leq -\beta^k \cdot \alpha \|\nabla f(x_i)\|^2 \}$$
$$f(x_i + \lambda h_i) - f(x_i) + \lambda \alpha \|\nabla f(x_i)\|^2 \leq 0 \text{ for } \lambda = \beta^k$$
$$\text{if } \lambda \in (0, \frac{2(1-\alpha)}{M}), \text{ then } f(x_i + \lambda h_i) - f(x_i) + \lambda \alpha \|\nabla f(x_i)\|^2 \leq 0$$

1.3 Step 3 - Update

Update $x_{i+1} = x_i + \lambda_i h_i$ replace i by i + 1 and goto Step 1.

$$f(x_i + \lambda h_i) - f(x_i) + \lambda \alpha \|\nabla f(x_i)\|^2 \leq \lambda \|\nabla f(x_i)\|^2 (\frac{M}{2} \lambda - (1 - \alpha))$$

To Calculate Armijo gradient algorithm when $x_0 = 0.5$, h_0 becomes -0.5451. [h1, h2, h3...]

= [0.02001627842295628, 2.658723154969067e07, 1.3633538742396922e12, 0.0, 0.0,, ,]

Eventually, h_i converges to be 0.

when $x_0 = 2$, [h1, h2, h3...] = [5.0693825587533325, 3.5393665775984573e07, 1.5543122344752192, 0.0, 0.0, , ,]

Eventually, h_i converges to be 0.